TURKISH JOURNAL OF SCIENCE
VOLUME 9, ISSUE 2, 91–100
ISSN: 2587–0971

https://www.tjoscience.com

An Improved Ratio Estimator for Estimating the Variance of Covid-19

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Abstract. Some rate-type proposed estimators of population variance, which use known skewness coefficients for auxiliary information in simple random sampling, are introduced. Ratio-type estimators combined by taking a weighted average of these proposed estimators are also presented. The mean square error (MSE) expressions of the proposed estimators are expressed up to the first order of approximation. After comparing MSEs of some competing estimators, it is shown theoretically that the proposed combining ratio estimators perform better than the unbiased estimator, and the estimators which were introduced in [3] and [4], and the proposed estimators for simple random sampling. In addition, the results are verified with the aid of Covid-19 datasets.

1. Introduction

Auxiliary information plays an important role in decreasing the variance of an estimator in sampling design. For the ratio estimator of the population variance, it is necessary to know the population variance of the auxiliary variable. Starting from here, most researchers have suggested many ratio estimators for effective estimation of population variance by taking advantage of the correlation between the auxiliary and study variables. Some of the studies in this vein of literature include, *inter alia*, [1, 2, 6–12, 14–17].

It is discussed the following usual unbiased estimator of variance (t_0=s_y^2) and some existing estimators of variance S_y^2. The usual unbiased estimator of variance is given by:

$$V(t_0) = MSE(t_0) = \frac{S_y^4}{n}(\beta_2(y) - 1) \tag{1}$$

When the population variance S_x^2 of the auxiliary variable x is known, the ratio estimator of the population variance S_y^2, according to [3], is given by:

$$t_R = s_y^2 \frac{S_x^2}{s_x^2}.$$

To the first degree of approximation, the MSE of the estimator t_R, given by:

$$MSE(t_R) = \frac{S_y^4}{n}[\beta_2(y) + \beta_2(x) - 2\theta] \tag{2}$$

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Received: 6 June 2024; Accepted: 21 August 2024; Published: 30 September 2024
Keywords. Auxiliary variables, finite population variance, efficiency, mean square error, Covid-19.
2010 *Mathematics Subject Classification*. 62D05, 62G05.
Cited this article as: Yücesoy, E. (2024) An Improved Ratio Estimator for Estimating the Variance of Covid-19. Turkish Journal of Science, 9(2), 91-100.

where $\beta_2(y)=\frac{\mu_{40}}{\mu_{20}^2}$ is the population coefficient of kurtosis of the studied variable. $\beta_2(x)=\frac{\mu_{04}}{\mu_{02}^2}$ is population coefficient of kurtosis of the auxiliary variable. $\theta=\frac{\mu_{22}}{\mu_{20}\mu_{02}}$ and $\mu_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\overline{Y}\right)^r\left(x_i-\overline{X}\right)^s$ with r,s being nonnegative integers [4].

Ratio estimator of the population variance S_y^2 due to [4] is given by:

$$t_{KC1}=\frac{s_y^2}{s_x^2-C_x}\left(S_x^2-C_x\right) \tag{3}$$

$$t_{KC2}=\frac{s_y^2}{s_x^2-\beta_2(x)}\left(S_x^2-\beta_2(x)\right) \tag{4}$$

$$t_{KC3}=\frac{s_y^2}{s_x^2\beta_2(x)-C_x}\left(S_x^2\beta_2(x)-C_x\right) \tag{5}$$

$$t_{KC4}=\frac{s_y^2}{s_x^2C_x-\beta_2(x)}\left(S_x^2C_x-\beta_2(x)\right) \tag{6}$$

where s_y^2 and s_x^2 are unbiased estimators of population variances S_y^2 and S_x^2, respectively. $C_x=\frac{S_x}{\overline{X}}$ is the coefficient of variation for the population and $\beta_2(x)$ is the population kurtosis of the auxiliary variable. The MSE equations of the estimator between Eq. (3)-(6) can be found using the Taylor series expansion [4]:

$$MSE\left(t_{KCi}\right)\cong\frac{S_y^4}{n}\left\{\beta_2(y)-1-2A_i(\theta-1)+A_i^2\left[\beta_2(x)-1\right]\right\};\; i=1,2,3,4 \tag{7}$$

where, $A_1=\frac{S_x^2}{S_x^2-C_x}$, $A_2=\frac{S_x^2}{S_x^2-\beta_2(x)}$, $A_3=\frac{S_x^2\beta_2(x)}{S_x^2\beta_2(x)-C_x}$ and $A_4=\frac{S_x^2C_x}{S_x^2C_x-\beta_2(x)}$.

2. Proposed Estimators

Following [4], it is suggested ratio-type estimators of the population variance S_y^2 using the coefficient of population skewness. These estimators are modified as below:

$$t_{SCi}=\frac{s_y^2}{m_1s_x^2-m_2}\left(m_1S_x^2-m_2\right); i=1,2,3,4,5$$

where, m_1 and m_2 are either real number of the function of the known parameter of auxiliary attributes such as C_x, $\beta_1(x)$, and $\beta_2(x)$.

Some of the rate type estimators of the population mean, which can be found by proper choice of constants m_1 and m_2 are shown in Table 1.

Here, $\beta_1(x)$ is the population skewness of the auxiliary variable, $\beta_2(x)$ is the population kurtosis of the auxiliary variable, and C_x is the coefficient of variation of the auxiliary variable.

It is defined that the MSE equation of these estimators is in the same form as the MSE Equation in (7). The MSE equations of these estimators are given as:

$$MSE\left(t_{SCi}\right)\cong\frac{S_y^4}{n}\left\{\beta_2(y)-1-2\delta_i(\theta-1)+\delta_i^2\left[\beta_2(x)-1\right]\right\};\; i=1,2,3,4,5 \tag{8}$$

Estimators	Values of	
	m_1	m_2
$t_{SC1} = \frac{s_y^2}{s_x^2-\beta_1(x)}(S_x^2 - \beta_1(x))$	1	$\beta_1(x)$
$t_{SC2} = \frac{s_y^2}{s_x^2\beta_2(x)-\beta_1(x)}(S_x^2\beta_2(x) - \beta_1(x))$	$\beta_2(x)$	$\beta_1(x)$
$t_{SC3} = \frac{s_y^2}{s_x^2C_x-\beta_1(x)}(S_x^2C_x - \beta_1(x))$	C_x	$\beta_1(x)$
$t_{SC4} = \frac{s_y^2}{s_x^2\beta_1(x)-C_x}(S_x^2\beta_1(x) - C_x)$	$\beta_1(x)$	C_x
$t_{SC5} = \frac{s_y^2}{s_x^2\beta_1(x)-\beta_2(x)}(S_x^2\beta_1(x) - \beta_2(x))$	$\beta_1(x)$	$\beta_2(x)$

Table 1: Suggested estimators for population variance based on coefficient of skewness

where, $\delta_1 = \frac{S_x^2}{S_x^2-\beta_1(x)}$, $\delta_2 = \frac{S_x^2\beta_2(x)}{S_x^2\beta_2(x)-\beta_1(x)}$, $\delta_3 = \frac{S_x^2C_x}{S_x^2C_x-\beta_1(x)}$, $\delta_4 = \frac{S_x^2\beta_1(x)}{S_x^2\beta_1(x)-C_x}$ and $\delta_5 = \frac{S_x^2\beta_1(x)}{S_x^2\beta_1(x)-\beta_2(x)}$. The function given in Eq. (8) is utilized to obtain the MSE values of the proposed estimators. In addition, by combining the ratio estimators for population variance given in Table 1, estimators using the style of estimators given in [5] are suggested. The general situation of the proposed estimators is;

$$t_{prj} = \omega \frac{s_y^2}{s_x^2 - \beta_1(x)} \left(S_x^2 - \beta_1(x)\right) + (1-\omega) \frac{s_y^2}{s_x^2 m_1 - m_2} \left(S_x^2 m_1 - m_2\right),\ j = 1,2,3,4$$

where ω is an optimal value that makes the MSE of t_{prj} ($j = 1,2,3,4$) minimum. By using the first degree of approximation in Taylor series technique, the MSE of the t_{prj} can be found by:

$$MSE\left(t_{prj}\right) \cong d \sum d' \tag{9}$$

where

$$d = \left[\left.\frac{\partial h(k,l)}{\partial k}\right|_{S_y^2, S_x^2} \quad \left.\frac{\partial h(k,l)}{\partial l}\right|_{S_y^2, S_x^2} \right]$$

and

$$\sum = \begin{bmatrix} \frac{S_y^4}{n}[\beta_2(y) - 1] & \frac{S_y^2}{n} S_x^2 (\theta - 1) \\ \frac{S_y^2}{n} S_x^2 (\theta - 1) & \frac{S_x^4}{n}[\beta_2(x) - 1] \end{bmatrix}$$

Here $h(k,l) = t_{prj}$. Accordingly, d is obtained by:

$$d = \left[1 - \frac{\left(S_y^2 m_1\right)}{\left(S_x^2 m_1 - m_2\right)} - \omega \frac{S_y^2 \beta_1(x) m_1 - S_y^2 m_2}{\left(S_x^2 - \beta_1(x)\right)\left(S_x^2 m_1 - m_2\right)} \right] \tag{10}$$

Utilizing equation (9), the MSE estimator of t_{prj} is obtained.

$$MSE\left(t_{prj}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y)-1\right]+\left(\frac{\left(S_y^2 m_1\right)}{\left(S_x^2 m_1-m_2\right)}+\omega\frac{S_y^2\beta_1(x)\,m_1-S_y^2 m_2}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2 m_1-m_2\right)}\right)^2\frac{S_x^4}{n}\left[\beta_2(x)-1\right]$$
$$-2\left(\frac{\left(S_y^2 m_1\right)}{\left(S_x^2 m_1-m_2\right)}+\omega\frac{S_y^2\beta_1(x)\,m_1-S_y^2 m_2}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2 m_1-m_2\right)}\right)\frac{S_y^2}{n}S_x^2(\theta-1),\ j=1,2,3,4$$

For $m_1=\beta_2(x),\ \ m_2=\beta_1(x)$) in Table 1 ($t_{SC1}$ and t_{SC2});

$$t_{pr1}=\omega\frac{s_y^2}{s_x^2-\beta_1(x)}\left(S_x^2-\beta_1(x)\right)+(1-\omega)\frac{s_y^2}{s_x^2\beta_2(x)-\beta_1(x)}\left(S_x^2\beta_2(x)-\beta_1(x)\right),$$

The expressions for the MSE of this estimator are given in following Eq (11):

$$MSE\left(t_{pr1}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y)-1\right]+\left(\frac{\left(S_y^2\beta_2(x)\right)}{\left(S_x^2\beta_2(x)-\beta_1(x)\right)}+\omega\frac{S_y^2\beta_1(x)\,\beta_2(x)-S_y^2\beta_1(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_2(x)-\beta_1(x)\right)}\right)^2\frac{S_x^4}{n}\left[\beta_2(x)-1\right]$$
$$-2\left(\frac{\left(S_y^2\beta_2(x)\right)}{\left(S_x^2\beta_2(x)-\beta_1(x)\right)}+\omega\frac{S_y^2\beta_1(x)\,\beta_2(x)-S_y^2\beta_1(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_2(x)-\beta_1(x)\right)}\right)\frac{S_y^2}{n}S_x^2(\theta-1) \quad (11)$$

For $m_1=C_x,\ \ m_2=\beta_1(x)$ in Table 1 (t_{SC1} and t_{SC3});

$$t_{pr2}=\omega\frac{s_y^2}{s_x^2-\beta_1(x)}\left(S_x^2-\beta_1(x)\right)+(1-\omega)\frac{s_y^2}{s_x^2C_x-\beta_1(x)}\left(S_x^2C_x-\beta_1(x)\right). \quad (12)$$

The expressions for the MSE of this estimator of Eq. (12) are computed as:

$$MSE\left(t_{pr2}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y)-1\right]+\left(\frac{\left(S_y^2C_x\right)}{\left(S_x^2C_x-\beta_1(x)\right)}+\omega\frac{S_y^2\beta_1(x)\,C_x-S_y^2\beta_1(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2C_x-\beta_1(x)\right)}\right)^2\frac{S_x^4}{n}\left[\beta_2(x)-1\right]$$
$$-2\left(\frac{\left(S_y^2C_x\right)}{\left(S_x^2C_x-\beta_1(x)\right)}+\omega\frac{S_y^2\beta_1(x)\,C_x-S_y^2\beta_1(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2C_x-\beta_1(x)\right)}\right)\frac{S_y^2}{n}S_x^2(\theta-1).$$

For $m_1=\beta_1(x)$, $m_2=C_x$ in Table 1 (t_{SC1} and t_{SC4});

$$t_{pr3}=\omega\frac{s_y^2}{s_x^2-\beta_1(x)}\left(S_x^2-\beta_1(x)\right)+(1-\omega)\frac{s_y^2}{s_x^2\beta_1(x)-C_x}\left(S_x^2\beta_1(x)-C_x\right). \quad (13)$$

The expressions for the MSE of this estimator of Eq (13) are computed in Eq (14):

$$MSE\left(t_{pr3}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y)-1\right]+\left(\frac{\left(S_y^2\beta_1(x)\right)}{\left(S_x^2\beta_1(x)-C_x\right)}+\omega\frac{S_y^2\beta_1(x)\,\beta_1(x)-S_y^2C_x}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_1(x)-C_x\right)}\right)^2\frac{S_x^4}{n}\left[\beta_2(x)-1\right]$$
$$-2\left(\frac{\left(S_y^2\beta_1(x)\right)}{\left(S_x^2\beta_1(x)-C_x\right)}+\omega\frac{S_y^2\beta_1(x)\,\beta_1(x)-S_y^2C_x}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_1(x)-C_x\right)}\right)\frac{S_y^2}{n}S_x^2(\theta-1), \quad (14)$$

For $m_1 = \beta_1(x)$, $m_2 = \beta_2(x)$) in Table 1 (t_{SC1} and t_{SC4});

$$t_{pr4} = \omega \frac{s_y^2}{s_x^2 - \beta_1(x)} \left(S_x^2 - \beta_1(x)\right) + (1-\omega) \frac{s_y^2}{s_x^2\beta_1(x) - \beta_2(x)} \left(S_x^2\beta_1(x) - \beta_2(x)\right), \tag{15}$$

The expressions for the MSE of this estimator of Eq (15) are computed in Eq (16):

$$\begin{aligned} MSE\left(t_{pr4}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y)-1\right] + \left(\frac{\left(S_y^2\beta_1(x)\right)}{\left(S_x^2\beta_1(x)-\beta_2(x)\right)} + \omega \frac{S_y^2\beta_1(x)\beta_1(x) - S_y^2\beta_2(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_1(x)-C_x\right)}\right)^2 \frac{S_x^4}{n}\left[\beta_2(x)-1\right] \\ -2\left(\frac{\left(S_y^2\beta_1(x)\right)}{\left(S_x^2\beta_1(x)-\beta_2(x)\right)} + \omega \frac{S_y^2\beta_1(x)\beta_1(x) - S_y^2\beta_2(x)}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2\beta_1(x)-\beta_2(x)\right)}\right) \frac{S_y^2}{n} S_x^2(\theta-1) \end{aligned} \tag{16}$$

The optimal value of ω to minimize Eq. (10) can be calculated by:

$$\frac{\partial}{\partial\omega} MSE\left(t_{prj}\right) = 0$$

$$\begin{aligned} 2\omega\frac{S_x^4}{n}\left[\beta_2(x)-1\right]\left(\frac{S_y^2\beta_1(x)m_1 - S_y^2 m_2}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2 m_1 - m_2\right)}\right)^2 - 2\left(\frac{S_y^2\beta_1(x)m_1 - S_y^2 m_2}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2 m_1 - m_2\right)}\right) \\ \left\{\frac{S_y^2}{n}S_x^2(\theta-1) - \left(\frac{\left(S_y^2 m_1\right)}{\left(S_x^2 m_1 - m_2\right)}\right)\frac{S_x^4}{n}\left[\beta_2(x)-1\right]\right\} = 0. \end{aligned}$$

$$\omega^* = \frac{\left\{\frac{S_y^2}{n}S_x^2(\theta-1) - \left(\frac{\left(S_y^2 m_1\right)}{\left(S_x^2 m_1 - m_2\right)}\right)\frac{S_x^4}{n}\left[\beta_2(x)-1\right]\right\}}{\frac{S_x^4}{n}\left[\beta_2(x)-1\right]\left(\frac{S_y^2\beta_1(x)m_1 - S_y^2 m_2}{\left(S_x^2-\beta_1(x)\right)\left(S_x^2 m_1 - m_2\right)}\right)}. \tag{17}$$

Considering the optimal values of ω^* in Eq (17), the minimum MSE of the proposed estimators is obtained. All proposed estimators have the same minimum mean square error as:

$$MSE_{min}\left(t_{prj}\right) \cong \frac{S_y^4}{n}\left[\beta_2(y) - 1 - \frac{(\theta-1)^2}{\left(\beta_2(x)-1\right)}\right];\ j = 1,2,3,4.$$

3. Efficiency Comparisons

In this section, the MSE of proposed combining estimator t_{pr} is compared with the MSEs of existing estimators discussed in the literature.

Condition (i): By using Eq. (1) and (17), if

$$MSE\left(t_0\right) > MSE_{min}\left(t_{prj}\right)$$

$$\frac{S_y^4}{n}\frac{(\theta-1)^2}{\left(\beta_2(x)-1\right)} > 0$$

Condition (ii): By using Eq (2) and (17), if

$$MSE\left(t_R\right) > MSE_{min}\left(t_{prj}\right)$$

$$\frac{S_y^4}{n}\left[\beta_2\left(x\right) - 2\theta + 1 + \frac{\left(\theta - 1\right)^2}{\left(\beta_2\left(x\right) - 1\right)}\right] > 0.$$

Condition (iii): By using Eq (7) and (17), if

$$MSE\left(t_{KCi}\right) > MSE_{min}\left(t_{prj}\right)$$

$$\frac{S_y^4}{n}\left[-2A_i\left(\theta - 1\right) + A_i^2\left(\beta_2\left(x\right) - 1\right) + \frac{\left(\theta - 1\right)^2}{\left(\beta_2\left(x\right) - 1\right)}\right] > 0.$$

Condition (iv): By using Eq (8) and (17), if

$$MSE\left(t_{SCi}\right) > MSE_{min}\left(t_{prj}\right)$$

$$\frac{S_y^4}{n}\left[-2\delta_i\left(\theta - 1\right) + \delta_i^2\left(\beta_2\left(x\right) - 1\right) + \frac{\left(\theta - 1\right)^2}{\left(\beta_2\left(x\right) - 1\right)}\right] > 0.$$

Using the above-mentioned conditions (i)-(iv), it is inferred that the proposed combining ratio-type estimators t_{pr} perform better than all other competing estimators mentioned above.

4. Numerical Illustrations

To test whether the theoretical results hold, the efficiency of several estimators was compared using Covid-19 datasets. This dataset includes the data of the Covid-19 Hazard & Exposure index, Development & Deprivation index, Covid-19 Lack of Coping Capacity index, and Health Conditions index [13].

4.1. Population I ([13])

The population consists of the Covid-19 Hazard & Exposure index and Development & Deprivation index of 190 countries. In this population, the primary variable is the Covid-19 Hazard & Exposure index data scaled from 0 to 10 (0 is low hazard& exposure, 10 is high hazard & exposure) which is based on sanitation, drinking water, hygiene, and population. In addition, the auxiliary attribute is the Development & Deprivation index scaled from 0 to 10 (0 is high development (low deprivation), 10 is low development (high deprivation)), which is based on the Human Development Index and the Multidimensional Poverty Index. In the light of this information, the variables are defined as following:

y : Covid-19 Hazard & Exposure
x : Development & Deprivation

$N = 191$, $\overline{Y} = 4.235$, $\overline{X} = 4.164$, $S_y = 1.593$, $S_x = 3.175$,
$\beta_2\left(y\right) = 2.101$, $\beta_2\left(x\right) = 1.712$,
$\beta_1\left(y\right) = 0.544$, $\beta_1\left(x\right) = 0.166$, $\theta = 1.166$, $\rho = 0.83$

	Population I			Population II		
Estimators	50	75	100	50	75	100
t_0	0.141685	0.094456	0.070842	0.401866	0.267910	0.200933
t_R	0.074821	0.049881	0.037411	0.945401	0.630267	0.472700
t_{KC1}	0.077460	0.051640	0.038730	1.353447	0.902298	0.676724
t_{KC2}	0.083720	0.055813	0.041860	5.628351	3.752234	2.814176
t_{KC3}	0.076161	0.050774	0.038081	1.037116	0.691411	0.518558
t_{KC4}	0.089444	0.059629	0.044722	3.882710	2.588473	1.941355
t_{SC1}	0.075634	0.050423	0.037817	1.380654	0.920436	0.690327
t_{SC2}	0.075271	0.050180	0.037635	1.041711	0.694474	0.520855
t_{SC3}	0.075932	0.050621	0.037966	1.309356	0.872904	0.654678
t_{SC4}	0.095002	0.063334	0.047501	1.267566	0.845044	0.633783
t_{SC5}	0.293717	0.195811	0.146858	3.495179	2.330119	1.747589
t_{pr}	0.073149	0.048766	0.036575	0.392597	0.261731	0.196298

Table 2: MSE values of different estimators for simple random sampling

4.2. *Population II ([13])*

The population consists of the Covid-19 Lack of Coping Capacity index and Health Conditions index of 190 countries. In population 2, the primary variable is the Covid-19 Lack of Coping Capacity index data scaled from 0 to 10 (0 is low lack, 10 is high lack), which is based on Institutionally index, Infrastructure index, International Health Regulations core capacity scores, and Operational readiness index. In addition, the auxiliary attribute is the Health Conditions index data scaled from 0 to 10 (0 is favorable conditions, 10 is unfavorable conditions), which is based on the Incidence of Tuberculosis, the Malaria incidence per 1,000 population at risk, and People requiring interventions against neglected tropical diseases. In the light of this information, the variables are defined as;

y : Covid-19 Lack of Coping Capacity
x : Health Conditions

$N = 191$, $\overline{Y} = 4.676$, $\overline{X} = 2.005$, $S_y = 1.972$, $S_x = 2.268$,
$\beta_2(y) = 2.330$, $\beta_2(x) = 3.334$,
$\beta_1(y) = -0.336$, $\beta_1(x) = 1.182$, $\theta = 1.268$, $\rho = 0.64$

Here, the theoretical results are supported by two real data applications to show the superiority of the proposed estimators. In the first application, the variable of the study is taken as the Covid-19 Hazard & Exposure index and the auxiliary variable is Development & Deprivation index. The correlation coefficient between the Covid-19 Hazard & Exposure index and Development & Deprivation index is 0.83. From this result, it is understood that there is a strong positive relationship between these two variables. Ratio estimation can be made using these two variables. Combining ratio-type estimators with a mean square error of minimum is suggested for the ratio estimation of the variance of the Covid-19 Hazard & Exposure index using the Development & Deprivation index auxiliary variable.

In the second application, in the same fashion, the variable of interest is taken as the Covid-19 Lack of Coping Capacity index and the auxiliary variable as Health Conditions index. The correlation coefficient between the Covid-19 Lack of Coping Capacity index and Health Conditions index is 0.64. From this result, it is understood that there is a strong positive relationship between these two variables. The estimator with the minimum means square error for the ratio estimation of the variance of the Covid-19 Lack of Coping Capacity index using the Health Conditions index auxiliary variable was verified. The sample size is taken as n= 50, 75, and 100. Table 2 exhibits that the performance of the proposed combining estimators t_{pr} is more efficient than the usual unbiased estimator t_0, and the estimator t_R of [3], and the estimators of [4], t_{KCi}, (i= 1, 2, 3, 4), and the proposed estimators t_{SCi}, (i= 1, 2, 3, 4, 5). This is an expected result when considering the efficiency comparisons in Section 3. The conditions (i)-(iv) are satisfied the following for the proposed combining ratio-type estimators for simple random sampling:

For Population I,

$$\frac{S_y^4}{n}\frac{(\theta-1)^2}{(\beta_2(x)-1)} = 0.068 > 0. \tag{18}$$

Condition (i) is satisfied as to Eq (18).

$$\frac{S_y^4}{n}\left[\beta_2(x) - 2\theta + 1 + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0016 > 0. \tag{19}$$

Condition (ii) is satisfied as to Eq (19).

$$\frac{S_y^4}{n}\left[-2A_1(\theta-1) + A_1^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0043 > 0, \tag{20}$$

$$\frac{S_y^4}{n}\left[-2A_2(\theta-1) + A_2^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.011 > 0, \tag{21}$$

$$\frac{S_y^4}{n}\left[-2A_3(\theta-1) + A_3^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0030 > 0, \tag{22}$$

$$\frac{S_y^4}{n}\left[-2A_4(\theta-1) + A_4^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0043 > 0. \tag{23}$$

Condition (iii) is satisfied as to Eq (20), (21), (22) and (23).

$$\frac{S_y^4}{n}\left[-2\delta_1(\theta-1) + \delta_1^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0025 > 0, \tag{24}$$

$$\frac{S_y^4}{n}\left[-2\delta_2(\theta-1) + \delta_2^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0021 > 0, \tag{25}$$

$$\frac{S_y^4}{n}\left[-2\delta_3(\theta-1) + \delta_3^2(\beta_2(x)-1) + \frac{(\theta-1)^2}{(\beta_2(x)-1)}\right] = 0.0028 > 0, \tag{26}$$

$$\frac{S_y^4}{n}\left[-2\delta_4(\theta-1)+\delta_4^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.0022>0. \tag{27}$$

Condition (iv) is satisfied as to Eq (24), (25), (26) and (27).

For Population II

$$\frac{S_y^4}{n}\frac{(\theta-1)^2}{(\beta_2(x)-1)}=0.0092>0. \tag{28}$$

Condition (i) is satisfied as to Eq (28).

$$\frac{S_y^4}{n}\left[\beta_2(x)-2\theta+1+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.553>0 \tag{29}$$

Condition (ii) is satisfied as to Eq (29).

$$\frac{S_y^4}{n}\left[-2A_1(\theta-1)+A_1^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.961>0, \tag{30}$$

$$\frac{S_y^4}{n}\left[-2A_2(\theta-1)+A_2^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=5.236>0, \tag{31}$$

$$\frac{S_y^4}{n}\left[-2A_3(\theta-1)+A_3^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.645>0, \tag{32}$$

$$\frac{S_y^4}{n}\left[-2A_4(\theta-1)+A_4^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=3.490>0. \tag{33}$$

Condition (iii) is satisfied as to Eq (30), (31), (32) and (33).

$$\frac{S_y^4}{n}\left[-2\delta_1(\theta-1)+\delta_1^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.988>0, \tag{34}$$

$$\frac{S_y^4}{n}\left[-2\delta_2(\theta-1)+\delta_2^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.649>0, \tag{35}$$

$$\frac{S_y^4}{n}\left[-2\delta_3(\theta-1)+\delta_3^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.917>0, \tag{36}$$

$$\frac{S_y^4}{n}\left[-2\delta_4(\theta-1)+\delta_4^2(\beta_2(x)-1)+\frac{(\theta-1)^2}{(\beta_2(x)-1)}\right]=0.875>0. \tag{37}$$

Condition (iv) is satisfied as to Eq (34), (35), (36) and (37).

5. Conclusion

I have improved the combination of rate type estimators to estimate population variance. First, the performances of these estimators are discussed theoretically. The conditions (i)-(iv) have occurred as outcomes of the theoretical comparison of the proposed combining ratio-type estimators with the competing estimators. Based on these results, it is determined that the proposed competing ratio-type estimator performs better than the competing estimators. That is, the proposed combining ratio-type estimator is stronger than all competing estimators for the two studied populations. For n= 50, 75, and 100, depending on the Development & Deprivation index, an effective estimator has been suggested that makes the mean square error value of the average of the Covid-19 Hazard & Exposure minimum with 0.073149, 0.048766, and 0.036575, respectively. These findings represent that the proposed combining estimator is successful in estimating the Covid-19 Hazard & Exposure. Similarly, for n= 50, 75, and 100, depending on the health conditions index, it is seen for the proposed estimator that the average value of error squares of the mean ratio estimation of the Covid-19 Lack of Coping Capacity is minimum with 0.392597, 0.261731, and 0.19298, respectively. These results indicate that the proposed combining estimator is successful in estimating the Covid-19 Lack of Coping Capacity. In addition, it is worth mentioning that there is a marked reduction in MSE values when the size of the sample increases.

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