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Invariant and Lacunary Invariant Statistical Convergence of Order η for Double Set Sequences

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Abstract. In this study, for double set sequences, we introduced the notions of invariant and lacunary invariant statistical convergence of order η $(0 < \eta \leq 1)$ in the Wijsman sense. Also, we investigated the inclusion relations between them.

1. Introduction

Long after the notion of convergence for double sequences was introduced by Pringsheim [12], this notion was respectively extended to the notions of statistical convergence, lacunary statistical convergence and double σ-convergent lacunary statistical sequence by Mursaleen and Edely [5], Patterson and Savaş [11] and Savaş and Patterson [13]. Recently, for double sequences, on two new convergence concepts called double almost statistical and double almost lacunary statistical convergence of order α were studied by Savaş [14, 15].

Over the years, on the various convergence notions for set sequences have been studied by many authors. One of them, discussed in this study, is the notion of convergence in the Wijsman sense [1, 2, 6]. Using the notions of statistical convergence, double lacunary sequence and invariant mean, this notion was extended to new convergence notions for double set sequences by some authors [7–9]. In [8], Nuray and Ulusu studied on the notions of invariant and lacunary invariant statistical convergence in the Wijsman sense for double set sequences.

In this paper, using order η, we studied on new convergence notions in the Wijsman sense for double set sequences.

More information on the notions of convergence for real or set sequences can be found in [3, 4, 6, 10, 16–20].

2. Definitions and Notations

Firstly, let us remind the basic notions that need for a better understanding of our study (see, [7–9, 11]).

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For a metric space (Y, d), $\mu(y, C)$ denote the distance from y to C where

$$\mu(y, C) := \mu_y(C) = \inf_{c \in C} d(y, c)$$

for any $y \in Y$ and any non-empty set $C \subseteq Y$.

For a non-empty set Y, let a function $g : \mathbb{N} \to P_Y$ (the power set of Y) is defined by $g(m) = C_m \in P_Y$ for each $m \in \mathbb{N}$. Then, the sequence $\{C_m\} = \{C_1, C_2, \ldots\}$, which is the codomain elements of g, is called set sequences.

Throughout this study, (Y, d) will be considered as a metric space and C, C_{mn} will be considered as any non empty closed subsets of Y.

A double set sequence $\{C_{mn}\}$ is called convergent to the set C in the Wijsman sense if each $y \in Y$,

$$\lim_{m,n \to \infty} \mu_y(C_{mn}) = \mu_y(C).$$

A double set sequence $\{C_{mn}\}$ is called statistically convergent to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,

$$\lim_{p,q \to \infty} \frac{1}{pq} \left| \left\{ (m, n) : m \leq p, n \leq q, \left| \mu_y(C_{mn}) - \mu_y(C) \right| \geq \xi \right\} \right| = 0.$$

A double sequence $\theta_2 = \{(j_u, k_v)\}$ is called a double lacunary sequence if there exist increasing sequences (j_u) and (k_v) of the integers such that

$$j_0 = 0, \ h_u = j_u - j_{u-1} \to \infty \ \text{ and } \ k_0 = 0, \ \bar{h}_v = k_v - k_{v-1} \to \infty \ \text{ as } \ u, v \to \infty.$$

In general, the following notations is used for any double lacunary sequence:

$$\ell_{uv} = j_u k_v, \ \ h_{uv} = h_u \bar{h}_v, \ \ I_{uv} = \left\{ (m, n) : j_{u-1} < m \leq j_u \ \textit{ and } \ k_{v-1} < n \leq k_v \right\},$$

$$q_u = \frac{j_u}{j_{u-1}} \ \text{ and } \ q_v = \frac{k_v}{k_{v-1}}.$$

Throughout this study, $\theta_2 = \{(j_u, k_v)\}$ will be considered as a double lacunary sequence.

A double set sequence $\{C_{mn}\}$ is called lacunary statistically convergent to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,

$$\lim_{u,v \to \infty} \frac{1}{h_{uv}} \left| \left\{ (m, n) \in I_{uv} : \left| \mu_y(C_{mn}) - \mu_y(C) \right| \geq \xi \right\} \right| = 0.$$

Let σ be a mapping such that $\sigma : \mathbb{N}^+ \to \mathbb{N}^+$ (the set of positive integers). A continuous linear functional ψ on ℓ_∞ is called an invariant mean (or a σ-mean) if it satisfies the following conditions:

1. $\psi(x_s) \geq 0$, when the sequence (x_s) has $x_s \geq 0$ for all s,
2. $\psi(e) = 1$, where $e = (1, 1, 1, \ldots)$ and
3. $\psi(x_{\sigma(s)}) = \psi(x_s)$ for all $(x_s) \in \ell_\infty$.

The mappings σ are assumed to be one to one and such that $\sigma^m(s) \neq s$ for all $m, s \in \mathbb{N}^+$, where $\sigma^m(s)$ denotes the m th iterate of the mapping σ at s. Thus ψ extends the limit functional on c, in the sense that $\psi(x_s) = \lim x_s$ for all $(x_s) \in c$.

A double set sequence $\{C_{mn}\}$ is called invariant statistically convergent to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,

$$\lim_{p,q\to\infty} \frac{1}{pq}\left|\left\{(m,n) : m \leq p, n \leq q, \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| = 0$$

uniformly in s, t.

The set of all invariant statistically convergent double set sequences in the Wijsman sense is denoted by $\left\{W_2S_\sigma\right\}$.

A double set sequence $\{C_{mn}\}$ is called lacunary invariant statistically convergent to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,

$$\lim_{u,v\to\infty} \frac{1}{h_{uv}}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| = 0$$

uniformly in s, t.

3. Main Results

In this section, for double set sequences, we introduced the notions of invariant and lacunary invariant statistical convergence of order η $(0 < \eta \leq 1)$ in the Wijsman sense. Also, we investigated the inclusion relations between them.

Definition 3.1. *The double set sequence $\{C_{mn}\}$ is invariant statistically convergent of order η to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,*

$$\lim_{p,q\to\infty} \frac{1}{(pq)^\eta}\left|\left\{(m,n) : m \leq p, n \leq q, \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| = 0$$

uniformly in s, t where $0 < \eta \leq 1$ and we denote this in $C_{mn} \overset{W_2S_\sigma^\eta}{\longrightarrow} C$ format.

Example 3.2. *Let $Y = \mathbb{R}^2$ and a double set sequence $\{C_{mn}\}$ be defined as following:*

$$C_{mn} := \begin{cases} \left\{(a,b) \in \mathbb{R}^2 : a^2 + (b+1)^2 = \frac{1}{mn}\right\} & ; \ \textit{if } m \textit{ and } n \textit{ are square integers} \\ \{(-1,0)\} & ; \ \textit{otherwise.} \end{cases}$$

In this case, the double set sequence $\{C_{mn}\}$ is invariant statistically convergent of order η $(0 < \eta \leq 1)$ to the set $C = \{(-1,0)\}$ in the Wijsman sense.

Remark 3.3. *For $\eta = 1$, the notion of invariant statistical convergence of order η in the Wijsman sense coincides with the notion of invariant statistical convergence in the Wijsman sense for double set sequences in [8].*

Definition 3.4. *The double set sequence $\{C_{mn}\}$ is lacunary invariant statistically convergent of order η to the set C in the Wijsman sense if every $\xi > 0$ and each $y \in Y$,*

$$\lim_{u,v\to\infty} \frac{1}{h_{uv}^\eta}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| = 0$$

uniformly in s, t where $0 < \eta \leq 1$ and we denote this in $C_{mn} \overset{W_2S_{\sigma\theta}^\eta}{\longrightarrow} C$ format.

The set of all lacunary invariant statistically convergent double set sequences of order η in the Wijsman sense is denoted by $\left\{W_2S_{\sigma\theta}^\eta\right\}$.

Example 3.5. *Let $Y = \mathbb{R}^2$ and a double set sequence $\{C_{mn}\}$ be defined as following:*

$$C_{mn} := \begin{cases} \left\{(a,b) \in \mathbb{R}^2 : (a+m)^2 + (b-n)^2 = 1\right\} & ; \text{ if } (m,n) \in I_{uv},\ m \text{ and } n \text{ are square integers} \\ \{(1,1)\} & ; \text{ otherwise.} \end{cases}$$

In this case, the double set sequence $\{C_{mn}\}$ is lacunary invariant statistically convergent of order η $(0 < \eta \leq 1)$ to the set $C = \{(1,1)\}$ in the Wijsman sense.

Remark 3.6. *For $\eta = 1$, the notion of lacunary invariant statistical convergence of order η in the Wijsman sense coincides with the notion of lacunary invariant statistical convergence in the Wijsman sense for double set sequences in [8].*

Theorem 3.7. *If*

$$\liminf_u q_u^\eta > 1 \ \text{ and } \ \liminf_v q_v^\eta > 1$$

where $0 < \eta \leq 1$, then

$$C_{mn} \stackrel{W_2 S_\sigma^\eta}{\longrightarrow} C \Rightarrow C_{mn} \stackrel{W_2 S_{\sigma\theta}^\eta}{\longrightarrow} C.$$

Proof. Let $0 < \eta \leq 1$ and suppose that $\liminf_u q_u^\eta > 1$ and $\liminf_v q_v^\eta > 1$. Then, there exist $\alpha, \beta > 0$ such that $q_u^\eta \geq 1 + \alpha$ and $q_v^\eta \geq 1 + \beta$ for all u, v, which implies that

$$\frac{h_{uv}}{\ell_{uv}} \geq \frac{\alpha\beta}{(1+\alpha)(1+\beta)} \Rightarrow \frac{h_{uv}^\eta}{\ell_{uv}^\eta} \geq \frac{\alpha^\eta \beta^\eta}{(1+\alpha)^\eta (1+\beta)^\eta}.$$

For every $\xi > 0$ and each $y \in Y$, we have

$$\begin{aligned} &\frac{1}{\ell_{uv}^\eta}\left|\left\{(m,n) : m \leq j_u, n \leq k_v, \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \\ &\geq \frac{1}{\ell_{uv}^\eta}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \\ &= \frac{h_{uv}^\eta}{\ell_{uv}^\eta}\frac{1}{h_{uv}^\eta}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \\ &\geq \frac{\alpha^\eta \beta^\eta}{(1+\alpha)^\eta (1+\beta)^\eta}\frac{1}{h_{uv}^\eta}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \end{aligned}$$

for all s, t. If $C_{mn} \stackrel{W_2 S_\sigma^\eta}{\longrightarrow} C$, then for each $y \in Y$ the term on the left side of the above inequality convergent to 0 and this implies that

$$\frac{1}{h_{uv}^\eta}\left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \to 0$$

uniformly in s, t. Thus, we get $C_{mn} \stackrel{W_2 S_{\sigma\theta}^\eta}{\longrightarrow} C$. □

Theorem 3.8. *If*

$$\limsup_u q_u < \infty \ \text{ and } \ \limsup_v q_v < \infty,$$

then

$$C_{mn} \stackrel{W_2 S_{\sigma\theta}^\eta}{\longrightarrow} C \Rightarrow C_{mn} \stackrel{W_2 S_\sigma^\eta}{\longrightarrow} C$$

where $0 < \eta \leq 1$.

Proof. Let $\limsup_u q_u < \infty$ and $\limsup_v q_v < \infty$. Then, there exist $M, N > 0$ such that $q_u < M$ and $q_v < N$ for all u, v. Also, we suppose that $C_{mn} \xrightarrow{W_2 S^{\eta}_{\sigma\theta}} C$ (where $0 < \eta \leq 1$) and $\xi > 0$, and let

$$\kappa_{uv} := \left|\left\{(m,n) \in I_{uv} : \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right|.$$

Then, there exist $u_0, v_0 \in \mathbb{N}$ such that for every $\xi > 0$, each $y \in Y$ and all $u \geq u_0, v \geq v_0$

$$\frac{\kappa_{uv}}{h^{\eta}_{uv}} < \xi$$

for all s, t. Now, let

$$\gamma := \max\{\kappa_{uv} : 1 \leq u \leq u_0,\ 1 \leq v \leq v_0\},$$

and let p and q be any integers satisfying $j_{u-1} < p \leq j_u$ and $k_{v-1} < q \leq k_v$. Then, for each $y \in Y$ we have

$$\begin{aligned}
&\frac{1}{(pq)^{\eta}}\left|\left\{(m,n) : m \leq p, n \leq q, \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \\
&\quad \leq \frac{1}{\ell^{\eta}_{(u-1)(v-1)}}\left|\left\{(m,n) : m \leq j_u, n \leq k_v, \left|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\right| \geq \xi\right\}\right| \\
&\quad = \frac{1}{\ell^{\eta}_{(u-1)(v-1)}}\left\{\kappa_{11} + \kappa_{12} + \kappa_{21} + \kappa_{22} + \cdots + \kappa_{u_0 v_0} + \cdots + \kappa_{uv}\right\} \\
&\quad \leq \frac{u_0 v_0}{\ell^{\eta}_{(u-1)(v-1)}}\left(\max_{\substack{1 \leq m \leq u_0 \\ 1 \leq n \leq v_0}}\{\kappa_{mn}\}\right) \\
&\qquad + \frac{1}{\ell^{\eta}_{(u-1)(v-1)}}\left\{h^{\eta}_{u_0(v_0+1)}\frac{\kappa_{u_0(v_0+1)}}{h^{\eta}_{u_0(v_0+1)}} + h^{\eta}_{(u_0+1)v_0}\frac{\kappa_{(u_0+1)v_0}}{h^{\eta}_{(u_0+1)v_0}}\right. \\
&\qquad\qquad \left. + h^{\eta}_{(u_0+1)(v_0+1)}\frac{\kappa_{(u_0+1)(v_0+1)}}{h^{\eta}_{(u_0+1)(v_0+1)}} + \cdots + h^{\eta}_{uv}\frac{\kappa_{uv}}{h^{\eta}_{uv}}\right\} \\
&\quad \leq \frac{u_0 v_0 \gamma}{\ell^{\eta}_{(u-1)(v-1)}} + \frac{1}{\ell^{\eta}_{(u-1)(v-1)}}\left(\sup_{\substack{u > u_0 \\ v > v_0}}\frac{\kappa_{uv}}{h^{\eta}_{uv}}\right)\left(\sum_{m,n \geq u_0, v_0}^{u,v} h^{\eta}_{mn}\right) \\
&\quad \leq \frac{u_0 v_0 \gamma}{\ell^{\eta}_{(u-1)(v-1)}} + \frac{1}{\ell_{(u-1)(v-1)}}\left(\sup_{\substack{u > u_0 \\ v > v_0}}\frac{\kappa_{uv}}{h^{\eta}_{uv}}\right)\left(\sum_{m,n \geq u_0, v_0}^{u,v} h_{mn}\right) \\
&\quad \leq \frac{u_0 v_0 \gamma}{\ell^{\eta}_{(u-1)(v-1)}} + \xi\frac{(j_u - j_{u_0})(k_v - k_{v_0})}{\ell_{(u-1)(v-1)}} \\
&\quad \leq \frac{u_0 v_0 \gamma}{\ell^{\eta}_{(u-1)(v-1)}} + \xi\, q_u\, q_v \\
&\quad \leq \frac{u_0 v_0 \gamma}{\ell^{\eta}_{(u-1)(v-1)}} + \xi M N
\end{aligned}$$

for all s, t. Since $j_{u-1}, k_{v-1} \to \infty$ as $p, q \to \infty$, it follows that for each $y \in Y$

$$\frac{1}{(pq)^{\eta}}\Big|\Big\{(m,n):\ m \le p, n \le q, \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}\Big| \to 0$$

uniformly in s, t. Thus, we get $C_{mn} \overset{W_2S_{\sigma}^{\eta}}{\longrightarrow} C$. $\square$

Theorem 3.9. *If*

$$1 < \liminf_u q_u^{\eta} \le \limsup_u q_u < \infty \ \ and \ \ 1 < \liminf_v q_v^{\eta} \le \limsup_v q_v < \infty$$

where $0 < \eta \le 1$, *then*

$$C_{mn} \overset{W_2S_{\sigma\theta}^{\eta}}{\longrightarrow} C \Leftrightarrow C_{mn} \overset{W_2S_{\sigma}^{\eta}}{\longrightarrow} C.$$

Proof. This can be obtained from Theorem 3.7 and Theorem 3.8, immediately. $\square$

Theorem 3.10. *If*

$$\liminf_{u,v\to\infty} \frac{h_{uv}^{\eta}}{\ell_{uv}} > 0$$

where $0 < \eta \le 1$, *then*

$$\Big\{W_2S_{\sigma}\Big\} \subseteq \Big\{W_2S_{\sigma\theta}^{\eta}\Big\}.$$

Proof. For every $\xi > 0$ and each $y \in Y$, it is obvious that

$$\Big\{(m,n): m \le j_u, n \le k_v, \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\} \supset \Big\{(m,n) \in I_{uv}: \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}.$$

Thus, we have

$$\begin{aligned}
&\frac{1}{\ell_{uv}}\Big|\Big\{(m,n):\ m \le j_u, n \le k_v, \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}\Big| \\
&\qquad \ge \frac{1}{\ell_{uv}}\Big|\Big\{(m,n) \in I_{uv}: \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}\Big| \\
&\qquad = \frac{h_{uv}^{\eta}}{\ell_{uv}} \frac{1}{h_{uv}^{\eta}}\Big|\Big\{(m,n) \in I_{uv}: \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}\Big|
\end{aligned}$$

for all s, t. If $C_{mn} \overset{W_2S_{\sigma}}{\longrightarrow} C$, then for each $y \in Y$ the term on the left side of the above inequality convergent to 0 and this implies that

$$\frac{1}{h_{uv}^{\eta}}\Big|\Big\{(m,n) \in I_{uv}: \big|\mu_y(C_{\sigma^m(s)\sigma^n(t)}) - \mu_y(C)\big| \ge \xi\Big\}\Big| \to 0$$

uniformly in s, t. Thus, we get $C_{mn} \overset{W_2S_{\sigma\theta}^{\eta}}{\longrightarrow} C$. Consequently,

$$\{W_2S_{\sigma}\} \subseteq \{W_2S_{\sigma\theta}^{\eta}\}.$$

$\square$

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