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Analysis of the spread of Hookworm infection with Caputo-Fabrizio fractional derivative

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Abstract. This research study provides a mathematical analysis for the spread of Hookworm infection model. Firstly, the proposed disease model is extended by means of the Caputo-Fabrizio fractional derivative. Then, existence and uniqueness of the solution is presented for the fractional-type Hookworm infection model with the help of the fixed-point theorem. Theoretical results of the model under consideration show the advantages of the fractional differential operators.

1. Introduction

In comparison to traditional mathematical models, fractional-order models are more advantageous since they generally produce better outcomes than classical order models [1]. Many researchers have concentrated on studying non-linear dynamical systems based on different types of fractional differential operators, inspired by the growth of fractional calculus, by creating a number of analytical or numerical techniques in order to obtain solutions [2, 3]. In order to analyze and investigate these systems, Riemann-Liouville (RL), Caputo, Caputo-Fabrizio (CF), Atangana-Baleanu (AB), as well as other non-local fractional derivatives, are employed to reach more detailed results. Recently, a new-type fractional derivative including a non-singular kernel has been presented as can be seen in [4]. The kernel of this non-local non-singular fractional operator has the form of the exponential function. Some type of fractional operators, on the other hand, have a power-law kernel and are limited in their ability to describe physical situations. Therefore, Caputo and Fabrizio proposed an additional fractional differential operator with an exponential decay kernel to overcome this challenge in [1]. The CF fractional derivative operator, which has a non-singular kernel, is a new approach to the fractional calculus that has captivated the interest of many researchers. Additionally, the CF operator is one of the best suited for simulating real-world problems that follow the exponential decay law. Developing a mathematical model employing the CF fractional-order derivative became a well-known subject of study over time [10–12].

Inspired by the above information, the Hookworm infection model [5] is investigated in this study utilizing CF fractional-type derivative and integral operator. First, the model is updated to use CF fractional operator. The existence and uniqueness of solutions are then determined under initial conditions utilizing the fixed point theorem.

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2. Preliminaries

In the current portion, some fundamental definitions of fractional derivative and integral are presented. For more information on fractional calculus, we refer the readers to [6–9].

Definition 2.1. *Let $n \in \mathbb{N}$ and $n-1 < \nu < n$, then Caputo fractional derivative is defined by [7]:*

$$ {}_a^C D_t^{\nu} f(t)) = \frac{1}{\Gamma(n-\nu)} \int_a^t \frac{f^{(n)}(r)}{(t-r)^{\nu+1-n}} dr. \tag{1}$$

Definition 2.2. *For $f \in H^1(a,b), b > a, \nu \in (0,1)$, the CF fractional derivative is presented as [4]:*

$$ {}_a^{CF}\mathfrak{D}_t^{\nu}(f(t)) = \frac{\nu M(\nu)}{1-\nu} \int_a^t \frac{df(x)}{dx} exp\Big[-\nu\frac{t-x}{1-\nu}\Big]dx. \tag{2}$$

Here $M(\nu)$ is a normalization constants given by $M(0) = M(1) = 1$. Also, the definition of CF operator can be given as below:

$$ {}_a^{CF}\mathfrak{D}_t^{\nu}(f(t)) = \frac{\nu M(\nu)}{1-\nu} \int_a^t \Big(f(t) - f(x)\Big) exp\Big[-\nu\frac{t-x}{1-\nu}\Big]dx.$$

Remark 2.3. *If $\eta = \frac{1-\nu}{\nu} \in (0,\infty), \nu = \frac{1}{1+\eta} =\in [0,1]$, then the above equation supposes the following expression*

$$ \mathfrak{D}_t^{\eta}(f(t)) = \frac{N(\eta)}{\eta} \int_a^t \frac{df(x)}{dx} exp\Big[-\frac{t-x}{\eta}\Big]dx, \quad N(0) = N(\infty) = 1. \tag{3}$$

Furthermore,

$$ \lim_{\nu\to 0} \frac{1}{\nu} exp\Big[-\frac{t-x}{\nu}\Big] = \delta(x-t).$$

It should be noted that according to the definition, the fractional integral of Caputo type function with order ν is an average between function f and its integral of order one. Hence, this means that

$$ M(\nu) = \frac{2}{2-\nu}, \quad 0 \le \nu \le 1. \tag{4}$$

Owing to the above expression, Nieto and Losada presented the new Caputo type derivative of order ν can be rewritten as follows:

Definition 2.4. *The fractional derivative of order ν is [6],*

$$ {}^{CF}\mathfrak{D}_{\star}^{\nu}(f(t)) = \frac{1}{1-\nu} \int_0^t f'(x) exp\Big[-\nu\frac{t-x}{1-\nu}\Big]dx. \tag{5}$$

At this instant subsequent to the preface of the novel derivative, the connected anti-derivative turns out to be imperative; the connected integral of the derivative was proposed by Nieto and Losada [6],

Definition 2.5. *Let $0 < \nu < 1.$, then the fractional integral with order ν of a function f is given by*

$$ {}^{CF}I^{\nu} f(t) = \frac{2(1-\nu)}{(2-\nu)M(\nu)} u(t) + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t u(s)ds, \qquad t \ge 0. \tag{6}$$

3. Fractional Model

In this section, we expand the spread of Hookworm infection model [5] to the fractional CF derivative. Classic integer order model is reformulated in the nonlinear system of differential in equations (7):

$$\begin{cases} \frac{S(t)}{dt} = \Lambda - \mu S(t)L_2(t) - \rho S(t) + \beta R(t) \\ \frac{E(t)}{dt} = \mu S(t)L_2(t) - \rho E(t) - \alpha\gamma E(t) - (1-\alpha)\gamma E(t) \\ \frac{I_1(t)}{dt} = (1-\alpha)\gamma E(t) - (\eta + \mu + \psi_1)I_1(t) \\ \frac{I_2(t)}{dt} = \alpha\gamma E(t) + \eta I_1(t) - (\mu + \rho + \psi_2)I_2(t) \\ \frac{R(t)}{dt} = \psi_1 I_1(t) + \psi_2 I_2(t) - (\rho + \beta)R(t) \\ \frac{F(t)}{dt} = \phi I_1(t) + \phi I_2(t) - (w + \chi)F(t) \\ \frac{L_1(t)}{dt} = \chi F(t) - (\delta + \zeta)L_1(t) \\ \frac{L_2(t)}{dt} = \zeta L_1(t) - kL_2(t). \end{cases} \tag{7}$$

In the above system (3.1), $S(t), E(t), I_1(t), I_2(t), R(t), F(t), L_1(t)$ and $L_2(t)$ represent the the dynamics of hookworm and human populations, susceptible humans,exposed humans, infective humans with moderate infection, infective humans with heavy infection, recovered humans and, worm eggs, non infective rhabditiform larvae, infective filariaform larvae respectively. All the parameters are positive constants and Λ is the recruited at the rate of the population, μ is the individuals from the recovery class at the rate, η is the moderate infectious individual progresses at the rate of the population, ψ_1 is the rate of recovery from moderate infection , ψ_2 is the rate of heavy infection, the natural death rate of human and the disease induced related mortality rate are denoted by ρ and μ while w, δ and k are respective death rates for eggs.

The spread of Hookworm infection model is integrated via Caputo–Fabrizio fractional derivative with the model and can be written as follows:

$$\begin{cases} {}_0^{CF}\mathfrak{D}_t^{\nu} S(t) = \Lambda - \mu S(t)L_2(t) - \rho S(t) + \beta R(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} E(t) = \mu S(t)L_2(t) - \rho E(t) - \alpha\gamma E(t) - (1-\alpha)\gamma E(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} I_1(t) = (1-\alpha)\gamma E(t) - (\eta + \mu + \psi_1)I_1(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} I_2(t) = \alpha\gamma E(t) + \eta I_1(t) - (\mu + \rho + \psi_2)I_2(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} R(t) = \psi_1 I_1(t) + \psi_2 I_2(t) - (\rho + \beta)R(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} F(t) = \phi I_1(t) + \phi I_2(t) - (w + \chi)F(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} L_1(t) = \chi F(t) - (\delta + \zeta)L_1(t), \\ {}_0^{CF}\mathfrak{D}_t^{\nu} L_2(t) = \zeta L_1(t) - kL_2(t). \end{cases} \tag{8}$$

where $\nu \in (0, 1)$ is the order of the fractional derivative operator. Then the initial values are as follows:

$$\begin{cases} S_{(0)}(t) = S(0), \quad E_{(0)}(t) = E(0), \quad I_{1_{(0)}}(t) = I_1(0), \\ I_{2_{(0)}}(t) = I_2(0), \quad R_{(0)}(t) = R(0), F_{(0)}(t) = F(0), \\ L_{1_{(0)}}(t) = L_1(0), \quad L_{2_{(0)}}(t) = L_2(0). \end{cases}$$

4. Existence and Uniqueness of Hookworm infection Model

Utilizing fixed point theorem, we show the existence of the model under investigation in this section. We utilize the CF integral operator on (9) in order to get

$$\begin{cases} S(t)-S(0) = {}_0^{CF} I_t^{\nu}\{\Lambda-\mu S(t)L_2(t)-\rho S(t)+\beta R(t)\},\\ E(t)-E(0) = {}_0^{CF} I_t^{\nu}\{\mu S(t)L_2(t)-\rho E(t)-\alpha\gamma E(t)-(1-\alpha)\gamma E(t)\},\\ I_1(t)-I_1(0) = {}_0^{CF} I_t^{\nu}\{(1-\alpha)\gamma E(t)-(\eta+\mu+\psi_1)I_1(t)\},\\ I_2(t)-I_2(0) = {}_0^{CF} I_t^{\nu}\{\alpha\gamma E(t)+\eta I_1(t)-(\mu+\rho+\psi_2)I_2(t)\},\\ R(t)-R(0) = {}_0^{CF} I_t^{\nu}\{\psi_1 I_1(t)+\psi_2 I_2(t)-(\rho+\beta)R(t)\}\\ F(t)-F(0) = {}_0^{CF} I_t^{\nu}\{\phi I_1(t)+\phi I_2(t)-(w+\chi)F(t)\},\\ L_1(t)-L_1(0) = {}_0^{CF} I_t^{\nu}\{\chi F(t)-(\delta+\zeta)L_1(t)\},\\ L_2(t)-L_2(0) = {}_0^{CF} I_t^{\nu}\{\zeta L_1(t)-kL_2(t)\}. \end{cases} \tag{9}$$

By using the approach in [6], we have

$$\begin{cases} S(t)-S(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\Lambda-\mu S(t)L_2(t)-\rho S(t)+\beta R(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\Lambda-\mu S(r)L_2(r)-\rho S(r)+\beta R(r)\}dr,\\ E(t)-E(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\mu S(t)L_2(t)-\rho E(t)-\alpha\gamma E(t)-(1-\alpha)\gamma E(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\mu S(r)L_2(r)-\rho E(r)-\alpha\gamma E(r)-(1-\alpha)\gamma E(r)\}dr,\\ I_1(t)-I_1(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{(1-\alpha)\gamma E(t)-(\eta+\mu+\psi_1)I_1(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{(1-\alpha)\gamma E(r)-(\eta+\mu+\psi_1)I_1(r)\}dr,\\ I_2(t)-I_2(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\alpha\gamma E(t)+\eta I_1(t)-(\mu+\rho+\psi_2)I_2(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\alpha\gamma E(r)+\eta I_1(r)-(\mu+\rho+\psi_2)I_2(r)\}dr,\\ R(t)-R(0) = \frac{2(1+\nu)}{(2-\nu)M(\nu)}\{\psi_1 I_1(t)+\psi_2 I_2(t)-(\rho+\beta)R(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\psi_1 I_1(r)+\psi_2 I_2(r)-(\rho+\beta)R(r)\}dr,\\ F(t)-F(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\phi I_1(t)+\phi I_2(t)-(w+\chi)F(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\phi I_1(r)+\phi I_2(r)-(w+\chi)F(r)\}dr,\\ L_1(t)-L_1(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\chi F(t)-(\delta+\zeta)L_1(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\chi F(r)-(\delta+\zeta)L_1(r)\}dr,\\ L_2(t)-L_2(0) = \frac{2(1-\nu)}{(2-\nu)M(\nu)}\{\zeta L_1(t)-kL_2(t)\}\\ \qquad +\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t\{\zeta L_1(r)-kL_2(r)\}dr. \end{cases} \tag{10}$$

For simplicity, we replace as follows:

$$\begin{cases} G_1(t,S) = \Lambda-\mu S(t)L_2(t)-\rho S(t)+\beta R(t),\\ G_2(t,E) = \mu S(t)L_2(t)-\rho E(t)-\alpha\gamma E(t)-(1-\alpha)\gamma E(t),\\ G_3(t,I_1) = (1-\alpha)\gamma E(t)-(\eta+\mu+\psi_1)I_1(t),\\ G_4(t,I_2) = \alpha\gamma E(t)+\eta I_1(t)-(\mu+\rho+\psi_2)I_2(t),\\ G_5(t,R)T = \psi_1 I_1(t)+\psi_2 I_2(t)-(\rho+\beta)R(t),\\ G_6(t,F) = \phi I_1(t)+\phi I_2(t)-(w+\chi)F(t),\\ G_7(t,L_1)T = \chi F(t)-(\delta+\zeta)L_1(t),\\ G_8(t,L_2) = \zeta L_1(t)-kL_2(t). \end{cases}$$

For proving our results, we assume the following assumption (H). For the following continuous functions $S(t), E(t), I_1(t), I_2(t), R(t), F(t), L_1(t), L_2(t) \in L[0,1]$, such that $\|S(t)\| \le c_1, \|E(t)\| \le c_2, \|I_1(t)\| \le c_3, \|I_2(t)\| \le c_4, \|R(t)\| \le c_5, \|F(t)\| \le c_6, \|L_1(t)\| \le c_7, \|L_2(t)\| \le c_8$.

Theorem 4.1. *The kernels $G_1, G_2, G_3, G_4, G_5, G_6, G_7$ and G_8 satisfy the Lipschitz condition if the assumption H is true and they are contractions provied that $\Phi_i < 1$ for $\forall \in i = 1, \dots, 8$.*

Proof. We start with G_1. Suppose that S and S_1 are two functions, then we obtain,

$$\begin{aligned}
\|G_1(t,S)-G_1(t,S_1)\| &= (\Lambda-\mu S(t)L_2(t)-\rho S(t)+\beta R(t))-(\Lambda-\mu S_1(t)L_2(t)-\rho S_1(t)+\beta R(t))\|.\\
&\le \{\mu L_2(t)+\rho\}\|(S(t)-S_1(t))\|\\
&\le \{\mu c_8+\rho\}\|(S(t)-S_1(t))\|\\
&\le \Phi_1\|(S(t)-S_1(t))\|.
\end{aligned}$$

Next, we prove for G_2. Suppose that E and E_1 are two functions, then we calculate in below,

$$\begin{aligned}
\|G_2(t,E)-G_2(t,E_1)\| &= (\mu S(t)L_2(t)-\rho E(t)-\alpha\gamma E(t)-(1-\alpha)\gamma E(t))\\
&\quad-(\mu S(t)L_2(t)-\rho E_1(t)-\alpha\gamma E_1(t)-(1-\alpha)\gamma E_1(t))\|.\\
&\le \{\rho+\alpha\gamma+(1-\alpha)\gamma\}\|(E(t)-E_1(t))\|\\
&\le \{\rho+1\}\|(E(t)-E_1(t))\|\\
&\le \Phi_2\|(E(t)-E_1(t))\|.
\end{aligned}$$

Then we show for G_3. Suppose that I_1 and I_{1_1} are two functions, then one can reach

$$\begin{aligned}
\|G_3(t,I_1)-G_3(t,I_{1_1})\| &= ((1-\alpha)\gamma E(t)-(\eta+\mu+\psi_1)I_1(t))\\
&\quad-((1-\alpha)\gamma E(t)-(\eta+\mu+\psi_1)I_{1_1}(t)))\|.\\
&\le \{(\eta+\mu+\psi_1)\}\|I_1(t)-I_{1_1}(t))\|\\
&\le \Phi_3\|I_1(t)-I_{1_1}(t))\|.
\end{aligned}$$

Similarly, we prove for G_4. Suppose that I_2 and I_{2_1} are two functions, then

$$\begin{aligned}
\|G_4(t,I_2)-G_4(t,I_{2_1})\| &= (\alpha\gamma E(t)+\eta I_1(t)-(\mu+\rho+\psi_2)I_2(t))\\
&\quad-(\alpha\gamma E(t)+\eta I_1(t)-(\mu+\rho+\psi_2)I_{2_1}(t))\|.\\
&\le \{(\mu+\rho+\psi_2)\}\|I_2(t)-I_{2_1}(t))\|\\
&\le \Phi_4\|I_2(t)-I_{2_1}(t))\|.
\end{aligned}$$

For G_5, we suppose that R and R_1 are two functions, then we have

$$\begin{aligned}
\|G_5(t,R)-G_5(t,R_1)\| &= (\psi_1 I_1(t)+\psi_2 I_2(t)-(\rho+\beta)R(t))\\
&\quad-(\psi_1 I_1(t)+\psi_2 I_2(t)-(\rho+\beta)R_1(t))\|.\\
&\le \{(\rho+\beta)\}\|(R(t)-R_1(t))\|\\
&\le \Phi_5\|(R(t)-R_1(t))\|.
\end{aligned}$$

Now suppose that F and F_1 are two functions, then for G_6 one can readily get

$$\begin{aligned}
\|G_6(t,F)-G_6(t,F_1)\| &= (\phi I_1(t)+\phi I_2(t)-(w+\chi)F(t))\\
&\quad-(\phi I_1(t)+\phi I_2(t)-(w+\chi)F_1(t))\|.\\
&\le \{(w+\chi)\}\|(F(t)-F_1(t))\|\\
&\le \Phi_6\|(F(t)-F_1(t))\|.
\end{aligned}$$

For G_7, supposing that L_1 and L_{1_1} are two functions, we can obtain

$$
\begin{aligned}
\|G_7(t,L_1)-G_3(t,L_{1_1})\| &= (\chi F(t)-(\delta+\zeta)L_1(t)) \\
&\quad -(\chi F(t)-(\delta+\zeta)L_1(t))\|. \\
&\le \{(\delta+\zeta)\}\|L_1(t)-L_{1_1}(t))\| \\
&\le \Phi_7\|L_1(t)-L_{1_1}(t))\|,
\end{aligned}
$$

and for G_8, suppose that L_2 and L_{2_1} are two functions, then we reach

$$
\begin{aligned}
\|G_8(t,L_2)-G_3(t,L_{2_1})\| &= (\zeta L_1(t)-kL_2(t)) \\
&\quad -(\zeta L_1(t)-kL_2(t))\|. \\
&\le \{(k)\}\|L_2(t)-L_{2_1}(t))\| \\
&\le \Phi_8\|L_2(t)-L_{2_1}(t))\|.
\end{aligned}
$$

All kernels which $G_i, i=1,\dots,8$ satisfy the conditions, so that they are contractions with $\Phi_i, i=1,\dots,8$. Therefore, this completes the proof.

Using notations for kernels, with all the initial values zero equation (9) becomes

$$
\begin{cases}
S(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_1(t,S)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_1(r,S))dr,\\
E(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_2(t,E)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_2(r,E))dr,\\
I_1(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_3(t,I_1)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_3(r,I_1))dr,\\
I_2(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_4(t,I_2)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_4(r,I_2))dr,\\
R(t)=\frac{2(1+\nu)}{(2-\nu)M(\nu)}G_5(t,R)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_5(r,R))dr,\\
F(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_6(t,F)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_6(r,F))dr,\\
L_1(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_7(t,L_1)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_7(r,L_1))dr,\\
L_2(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_8(t,L_2)+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_8(r,L_2))dr,
\end{cases}
$$

The following recursive formula is presented:

$$
\begin{cases}
S_n(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_1(t,S_{n-1})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_1(r,S_{n-1}))dr,\\
E_n(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_2(t,E_{n-1})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_2(r,E_{n-1}))dr,\\
I_{1(n)}(t)=\frac{2(1+\nu)}{(2-\nu)M(\nu)}G_3(t,I_{1(n-1)})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_3(r,I_{1(n-1)}))dr,\\
I_{2(n)}(t)=\frac{2(1+\nu)}{(2-\nu)M(\nu)}G_4(t,I_{2(n-1)})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_4(r,I_{2(n-1)}))dr,\\
R_n(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_5(t,R_{n-1})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_5(r,R_{n-1}))dr,\\
F_n(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_6(t,F_{n-1})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_6(r,F_{n-1}))dr,\\
L_{1(n)}(t)=\frac{2(1+\nu)}{(2-\nu)M(\nu)}G_7(t,L_{1(n-1)})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_7(r,L_{1(n-1)}))dr,\\
L_{2(n)}(t)=\frac{2(1-\nu)}{(2-\nu)M(\nu)}G_8(t,L_{2(n-1)})+\frac{2\nu}{(2-\nu)M(\nu)}\int_0^t(G_8(r,L_{2(n-1)}))dr.
\end{cases}
\tag{11}
$$

and

$$
\begin{cases}
S_{(0)}(t)=S(0),\\
E_{(0)}(t)=E(0),\\
I_{1(0)}(t)=I_1(0),\\
I_{2(0)}(t)=I_2(0),\\
R_{(0)}(t)=R(0),\\
F_{(0)}(t)=F(0),\\
L_{1(0)}(t)=L_1(0),\\
L_{2(0)}(t)=L_2(0).
\end{cases}
$$

where $S_{(0)}(t), E_{(0)}(t), M_{(0)}(t), I_{1(0)}(t), I_{2(0)}(t), R_{(0)}(t), F_{(0)}(t), L_{1(0)}(t)$ and $L_{2(0)}(t)$ are the initial conditions. The difference of the succeeding terms is obtained as

$$\begin{cases}
\Psi_{1n}(t) & = S_n(t) - S_{n-1}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t, S_{n-1}) - G_1(t, S_{n-2}) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, S_{n-1}) - G_1(r, S_{n-2})dr, \\
\Psi_{2n}(t) & = E_n(t) - E_{n-1}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_2(t, E_{n-1}) - G_2(t, E_{n-2}) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_2(r, E_{n-1}) - G_2(r, E_{n-2})dr, \\
\Psi_{3n}(t) & = I_{1(n)}(t) - I_{1(n-1)}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_3(t, I_{1(n-1)}) - G_3(t, I_{1(n-2)})) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_3(r, I_{1(n-1)}) - G_3(r, I_{1(n-2)})dr, \\
\Psi_{4n}(t) & = I_{2(n)}(t) - I_{2(n-1)}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_4(t, I_{2(n-1)}) - G_4(t, I_{2(n-2)})) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_4(r, I_{2(n-1)}) - G_4(r, I_{2(n-2)})dr, \\
\Psi_{5n}(t) & = R_n(t) - R_{n-1}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_5(t, R_{n-1}) - G_5(t, R_{n-2}) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_5(r, R_{n-1}) - G_5(r, R_{n-2})dr, \\
\Psi_{6n}(t) & = F_n(t) - F_{n-1}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_6(t, F_{n-1}) - G_6(t, F_{n-2}) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_6(r, F_{n-1}) - G_6(r, F_{n-2})dr, \\
\Psi_{7n}(t) & = L_{1(n)}(t) - L_{1(n-1)}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_7(t, L_{1(n-1)}) - G_7(t, L_{1(n-2)})) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_7(r, L_{1(n-1)}) - G_7(r, L_{1(n-2)})dr, \\
\Psi_{8n}(t) & = L_{2(n)}(t) - L_{2(n-1)}(t) \\
& = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_8(t, L_{2(n-1)}) - G_8(t, L_{2(n-2)})) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_8(r, L_{2(n-1)}) - G_8(r, L_{2(n-2)})dr.
\end{cases}$$

Notive that

$$\begin{cases}
S_n(t) = \sum_{i=1}^n \Psi_{1i}(t), \\
E_n(t) = \sum_{i=1}^n \Psi_{2i}(t), \\
I_{1(n)}(t) = \sum_{i=1}^n \Psi_{3i}(t), \\
I_{2(n)}(t) = \sum_{i=1}^n \Psi_{4i}(t), \\
R_n(t) = \sum_{i=1}^n \Psi_{5i}(t), \\
F_n(t) = \sum_{i=1}^n \Psi_{6i}(t), \\
L_{1(n)}(t) = \sum_{i=1}^n \Psi_{7i}(t), \\
L_{2(n)}(t) = \sum_{i=1}^n \Psi_{8i}(t).
\end{cases}$$

Now we continue the same process and we have the following form,

$$\begin{cases}
\|\Psi_{1n}(t)\| & = \|S_n(t) - S_{n-1}(t)\| \\
& = \|\frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t, S_{n-1}) - G_1(t, S_{n-2}) \\
& \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r, S_{n-1}) - G_1(r, S_{n-2})dr\|.
\end{cases}$$

Using the triangular inequality, equation (11) is simplified to

$$\begin{cases} ||S_n(t) - S_{n-1}(t)|| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}||(G_1(t,S_{n-1}) - G_1(t,S_{n-2})|| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}||\int_0^t (G_1(r,S_{n-1}) - G_1(r,S_{n-2})dr||. \end{cases}$$

Because of the fact that the kernel satisfyies the Lipschitz condition, then we can get

$$\begin{cases} ||S_n(t) - S_{n-1}(t)|| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_1||S_{n-1} - S_{n-2}|| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_1||\int_0^t ||S_{n-1} - S_{n-2}||dr. \end{cases} \tag{12}$$

Then we have

$$||\Psi_{1n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_1||\Psi_{1(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_1\int_0^t ||\Psi_{1(n-1)}(r)||dr.$$

Accordingly, we attain the results as below:

$$\begin{cases} ||\Psi_{2n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_2||\Psi_{2(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_2\int_0^t ||\Psi_{2(n-1)}(r)||dr, \\ ||\Psi_{3n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_3||\Psi_{3(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_3\int_0^t ||\Psi_{3(n-1)}(r)||dr, \\ ||\Psi_{4n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_4||\Psi_{4(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_4\int_0^t ||\Psi_{4(n-1)}(r)||dr, \\ ||\Psi_{5n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_5||\Psi_{5(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_5\int_0^t ||\Psi_{5(n-1)}(r)||dr, \\ ||\Psi_{6n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_6||\Psi_{6(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_6\int_0^t ||\Psi_{6(n-1)}(r)||dr, \\ ||\Psi_{7n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_7||\Psi_{7(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_7\int_0^t ||\Psi_{7(n-1)}(r)||dr, \\ ||\Psi_{8n}(t)|| \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_8||\Psi_{8(n-1)}(t)|| + \frac{2\nu}{(2-\nu)M(\nu)}\Phi_8\int_0^t ||\Psi_{8(n-1)}(r)||dr. \end{cases}$$

We shall then state the following theorem. □

Theorem 4.2. *The Hookworm infection model (9) has unique solution if the conditions below hold.*

$$\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_1 - \frac{2\nu}{(2-\nu)M(\nu)}\Phi_1 t < 1.$$

Proof. Since all the functions $S(t), E(t), I_1(t), I_2(t), R(t), F(t), L_1(t)$ and $L_2(t)$ are bounded, we can say that the kernels satisfy the Lipschitz condition, so by using the recursive method, we get the succeeding relation as

$$\begin{cases} ||\Psi_{1n}(t)|| \leq ||S_n(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_1\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_1 t\right)\right]^n, \\ ||\Psi_{2n}(t)|| \leq ||E_n(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_2\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_2 t\right)\right]^n, \\ ||\Psi_{3n}(t)|| \leq ||I_{1(n)}(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_3\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_3 t\right)\right]^n, \\ ||\Psi_{4n}(t)|| \leq ||I_{2(n)}(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_4\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_4 t\right)\right]^n, \\ ||\Psi_{5n}(t)|| \leq ||R_n(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_5\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_5 t\right)\right]^n, \\ ||\Psi_{6n}(t)|| \leq ||F_n(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_6\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_6 t\right)\right]^n, \\ ||\Psi_{7n}(t)|| \leq ||L_{1(n)}(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_7\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_7 t\right)\right]^n, \\ ||\Psi_{8n}(t)|| \leq ||L_{2(n)}(0)||\left[\left(\frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_8\right) + \left(\frac{2\nu}{(2-\nu)M(\nu)}\Phi_8 t\right)\right]^n. \end{cases} \tag{13}$$

Thus, the existence and continuity of the solutions is proved. Moreover, in order to ensure that the above function is a solution of equation (9), we continue as below:

$$\begin{cases} S(t) - S(0) = S_n(t) - A_n(t), \\ E(t) - E(0) = E_n(t) - B_n(t), \\ I_1(t) - I_1(0) = I_{1n}(t) - C_n(t), \\ I_2(t) - I_2(0) = T_{2n}(t) - D_n(t). \\ R(t) - R(0) = R_n(t) - G_n(t), \\ F(t) - F(0) = F_n(t) - H_n(t), \\ L_1(t) - L_1(0) = L_{1n}(t) - M_n(t), \\ L_2(t) - L_2(0) = L_{2n}(t) - N_n(t). \end{cases} \tag{14}$$

Therefore, we have

$$\begin{cases} \|A_n(t)\| & = \|\frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t,S_n) - G_1(t,S_{n-1}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r,S_n) - G_1(r,S_{n-1}))dr\| \\ & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)} \|(G_1(t,S_n) - G_1(t,S_{n-1}))\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t \|(G_1(r,S_n) - G_1(r,S_{n-1}))\|dr \\ & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)} \Phi_1 \|K - K_{n-1}\| + \frac{2\nu}{(2-\nu)M(\nu)} \Phi_1 \|S - S_{n-1}\| t. \end{cases}$$

Using the process in a recursive manner gives

$$\|A_n(t)\| \leq \left(\frac{2(1-\nu)}{(2-\nu)M(\nu)} + \frac{2\nu}{(2-\nu)M(\nu)} t \right)^{n-1} \Phi_1^{n+1} a. \tag{15}$$

By applying the limit on equation (4.8) as n tends to infinity, we get

$$\|A_n(t)\| \to 0.$$

Similarly,

$$\|B_n(t)\| \to 0, \quad \|C_n(t)\| \to 0, \quad \|D_n(t)\| \to 0,$$

$$\|G_n(t)\| \to 0, \quad \|H_n(t)\| \to 0, \|M_n(t)\| \to 0, \|N_n(t)\| \to 0$$

For the uniqueness system (9) solution, we take on contrary that there exists another solution of (9) given by $S_1(t), E_1(t), I_{11}(t), I_{12}(t), R_1(t), F_1(t), L_{11}(t)$ and $L_{12}(t)$. Then

$$\begin{cases} S(t) - S_1(t) & = \frac{2(1-\nu)}{(2-\nu)M(\nu)}(G_1(t,S_n) - G_1(t,S_{n-1}) \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t (G_1(r,S_n) - G_1(r,S_{n-1}))dr. \end{cases} \tag{16}$$

Taking norm on equation (16), we get

$$\begin{cases} \|S(t) - S_1(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)} \|(G_1(t,S_n) - G_1(t,S_{n-1})\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t \|(G_1(r,S_n) - G_1(r,S_{n-1}))\|dr. \end{cases}$$

If we apply the Lipschitz condition of kernel, we have

$$\begin{cases} \|S(t) - S_1(t)\| & \leq \frac{2(1-\nu)}{(2-\nu)M(\nu)} \Phi_1 \|S(t) - S_1(t)\| \\ & \quad + \frac{2\nu}{(2-\nu)M(\nu)} \int_0^t \Phi_1 t \|S(t) - S_1(t)\|dr. \end{cases}$$

It gives

$$\|S(t) - S_1(t)\| \left(1 - \frac{2(1-\nu)}{(2-\nu)M(\nu)} \Phi_1 - \frac{2\nu}{(2-\nu)M(\nu)} \Phi_1 t \right) \leq 0. \tag{17}$$

□

Theorem 4.3. *The model (9) solution will be unique if*

$$\left(1 - \frac{2(1-\nu)}{(2-\nu)M(\nu)}\Phi_1 - \frac{2\nu}{(2-\nu)M(\nu)}\Phi_1 t\right) > 0. \tag{18}$$

Proof. If condition (18) holds, then (17) implies that

$$\|S(t) - S_1(t)\| = 0.$$

Hence, we can attain

$$S(t) = S_1(t).$$

On employing the same procedure, we get

$$\begin{cases} E(t) = E_1(t), \\ I_1(t) = I_{11}(t), \\ I_2(t) = I_{21}(t). \\ R(t) = R_1(t), \\ F(t) = F_1(t), \\ L_1(t) = L_{11}(t), \\ L_2(t) = L_{21}(t). \end{cases}$$

□

5. Conclusion

The Hookworm infection model is analyzed employing the fractional derivative and integral operator presented by Caputo and Fabrizio. First, the model revised to the fractional derivative of Caputo–Fabrizio. Then, using the fixed point theorem, existence and uniqueness solutions were performed under initial conditions.

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