Bipolar Soft Expert Sets on Nearness Approximation Space

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Abstract. Bipolar soft set is provided with two soft sets, one positive and the other negative. By adding the opinions of experts to this set, it is easier to choose which feature is stronger to find the object we want. The concept of near soft set is a concept that can distinguish soft sets with similar properties according to the same equivalence class. These two sets of features and expert opinions can be thought of as a single feature set. In this article, the concept of a near bipolar soft expert set and its basic properties are introduced, to which the near set features are added to the bipolar soft expert set. In this new set, with the help of features and expert opinions, a new equivalence relation is created and objects are classified. Accordingly, the basic properties of the set can be examined. With the near bipolar soft expert set, practicality will be provided in decision making so that we can find the most suitable object in practice. This new idea is illustrated with real-life examples. Thanks to the near soft expert set, we make it easy to choose the one closest to the criteria we want in decision making. Among the many given objects, we can find the ones with similar properties with the properties we want with an equivalence relation that we restrict more by using the experts' opinion.

1. Introduction

Uncertainties are used to perform data analysis correctly. Some mathematical models were used to remove the uncertainty. In order to analyze the data close to the desired one, digital topology has been used [1] and many set types have been discussed in the literature. For this, different set concepts have been created. For example; with the help of objects and features on these objects, Pawlak [2] first presented the concept of rough set and then Peters [3, 4] presented the concept of near set, in which he examined sets close to each other with these features. Another set, the soft set, was created by Molodtsov [5] and has been studied by many people both in practice and in theory [6–11]. Feng and Li [12], on the other hand, established a new notion by integrating the concepts of soft set and rough set. One of the different versions of soft sets is described by Alkhazaleh and Salleh[13]. this kind of set determines the decision-making process with a group of experts[14]. After Tasbozan [15] combined the concepts of near and soft set. These concepts have been developed and produced in the topology [16, 17].

Bipolar soft set, which is a decision-making approach, has been defined [18–21] in order to make the most accurate object selection with the parameters determined by the decision maker with positive or negative features. This bipolar soft set model, created by incorporating the idea of bipolarity into the solution of ambiguous information, and succeeded in attracting the attention of many types of research [22, 23]. By

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limiting this information to the selected parameters, we obtained the notion of a near bipolar soft expert set in order to distinguish those with similar characteristics more quickly with the help of experts. With the new equivalence classes that we will obtain with approaches near to bipolar soft expert sets, practicality can be achieved in decision-making so that we can find the object we will choose in practice. In today's evaluation system, bipolar theory is used to understand people's positive or negative thoughts about objects. Adding expert opinions to this will enable us to find the result easier and more effective. In this way, organizations can track how much their products are liked by which experts, or they can help buyers find the products closest to their needs with the help of experts. Among these evaluations, the most suitable product can be selected depending on the features and expert opinion.

In this article, we introduced the concept of the near bipolar soft expert set. In other words, in the bipolar soft expert set, expert opinion was added to the bipolarity of knowledge, and the objects were grouped in this equivalence class by considering them as a single parameter. Accordingly, the selections in the group approved by the experts will be easily seen with the restricted features. An application showing how these situations can be expressed is given in the study.

2. Preliminary

Let *O* be an objects set, \mathcal{F} be a set of parameters that define properties on objects and 2^O is the set of all subsets of *O*, $o = \{0, 1\}$ be a set of opinions, *E* be a set of experts, $Z = B \times E \times o$ and $K \subseteq Z$.

Definition 2.1. Let $B \subseteq \mathcal{F}$ and $F : B \to 2^O$, then (F, B) is a soft set(SS) over O[7].

Definition 2.2. Let $NAS = (O, \mathcal{F}, \sim_{Br}, N_r, v_{N_r})$ be a nearness approximation space, B be a non-empty subsets of \mathcal{F} and (F, B) be a SS over O. Then

$$N_r * ((F, B)) = (N_r * (F(\phi) = \cup \{x \in O : [x]_{Br} \subseteq F(\phi)\}, B))$$

and

$$N_r^*((F,B)) = (N_r^*(F(\phi) = \bigcup \{x \in O : [x]_{Br} \cap F(\phi) \neq \emptyset\}, B))$$

are lower and upper near approximation operators.

The SS $N_r((F, B))$ with $Bnd_{N_r(B)}((F, B)) \ge 0$ is called a near soft set(NSS) [15].

Definition 2.3. Let $F : B \to P(O)$ and $G : \neg B \to P(O)$ be mappings which $F(\phi) \cap G(\neg \phi) = \emptyset$, $\forall \phi \in B$. $\sigma = (F, G, B)$ is called a bipolar soft set (BSS) over O[18].

Definition 2.4. A triplet (F, G, K) is called a bipolar soft expert set(BSES) over O, where F, G are mappings given by $F: B \rightarrow 2^O, G: -B \rightarrow 2^O$ such that

$$F(\phi, e, 1) \cap G(-\phi, e, 1) = \emptyset,$$

$$F(\phi, e, 0) \cap G(-\phi, e, 0) = \emptyset$$

for all $(\phi, e, o) \in K$ *and* $(-\phi, e, o) \in K$ [19].

3. Bipolar Soft Expert Sets on Nearness Approximation Space

Definition 3.1. Let O be an objects set, $B \subseteq \mathcal{F}, \mathcal{F}$ be a set of parameters that define properties on objects and 2^O is the power set of O, (F, B) be a SS, $o = \{0, 1\}$ be a set of opinions, E be a set of experts, $Z = B \times E \times o$ and $K \subseteq Z$. Let $N_r(\sigma)$ be a NSS and $\sigma = (F, G, K)$ be a BSES over O. Then the triplet $N_r(F, G, B)$ is called a BSES on nearness approximation space (BSENAS) over O, where F, G are mappings given by $F : B \to 2^O, G : \neg B \to 2^O$ such that

$$\begin{aligned} F(\phi, e, 1) \cap G(\neg \phi, e, 1) &= \emptyset, \\ F(\phi, e, 0) \cap G(\neg \phi, e, 0) &= \emptyset \end{aligned}$$

for all $(\phi, e, o) \in K$, $(\neg \phi, e, o) \in K$ and the equivalence classes obtained with the expert opinions about the properties of the objects are denoted by $[x]_{Br,e_i,1}$.

Then,

$$\begin{split} N_r * ((F, G, K)) &= (N_r * (F(\phi, e, o) = \cup \{ x \in O : [x]_{Br, e_i, 1} \subseteq F(\phi, E, o) \}), \\ N_r^* ((F, G, K)) &= (N_r^* (F(\phi, e, o) = \cup \{ x \in O : [x]_{Br, e_i, 1} \cap F(\phi, E, o) \neq \emptyset \}) \end{split}$$

are lower and upper near approximation operators. The BSES, $N_r((F, G, K))$ with $Bnd_{N_r(B)}((F, G, K)) \ge 0$ called a near bipolar soft expert set(NBSES).

For simplify in this paper, we assume that $o = \{1, 0\} = \{agree, disagree\}$ two-valued opinions, but multi-valued opinions may be used.

Example 3.2. Let $O = \{y_1, y_2, y_3, y_4, y_5\}$ be a five car and $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ be a set of parameters, where $\phi_1, \phi_2, \phi_3, \phi_4$ strong, fuel saving, speed and engine capacity respectively. The equivalence classes obtained with the properties of the objects ($\phi_i, i = 1, 2, 3, 4$) are given as follows:

 $\begin{array}{lll} [y_1]_{\phi_1} &=& \{y_1, y_3\}, [y_2]_{\phi_1} = \{y_2, y_4, y_5\}, \\ [y_1]_{\phi_2} &=& \{y_1, y_3\}, [y_2]_{\phi_2} = \{y_2, y_4\}, [y_5]_{\phi_2} = \{y_5\}. \end{array}$

 $F: B \to 2^O$ and $G: -B \to 2^O$ mappings given as follows:

$F(\phi_1, e_1, 1)$	=	$\{y_1, y_4\}, G(\neg \phi_1, e_1, 1) = \{y_3\},\$
$F(\phi_1,e_2,1)$	=	$\{y_1, y_3\}, G(\neg \phi_1, e_2, 1) = \{y_2\},$
$F(\phi_2,e_1,1)$	=	$\{y_3,y_5\}, G(\neg\phi_2,e_1,1)=\emptyset,$
$F(\phi_2,e_2,1)$	=	$\{y_1, y_4\}, G(\neg \phi_2, e_2, 1) = \{y_3\},$
$F(\phi_1,e_1,0)$	=	$\{y_2,y_3\}, G(\neg\phi_1,e_1,0)=\{y_1,y_4\},$
$F(\phi_1,e_2,0)$	=	$\{y_2, y_4\}, G(\neg \phi_1, e_2, 0) = \{y_1, y_3, y_5\},$
$F(\phi_2,e_1,0)$	=	$\{y_1, y_2\}, G(\neg \phi_2, e_1, 0) = \{y_3, y_4\},$
$F(\phi_2, e_2, 0)$	=	$\{y_2, y_3, y_5\}, G(\neg \phi_2, e_2, 0) = \{y_1, y_4\}$

and the equivalence classes obtained with the expert opinions about the properties of the objects are given as follows:

Then,

$$N(F, G, K) = \{((\phi_1, e_2, 1), \{y_1, y_3\}), ((\neg \phi_1, e_2, 1), \{y_2\}), ((\phi_2, e_2, 0), \{y_2, y_3, y_5\}), ((\neg \phi_2, e_2, 0), \{y_1, y_4\})\}$$

is a *NBSES*. Because, for $\phi_1, \phi_2 \in B$,

 $N_*((F,G,K))) = ([y_1]_{\phi_1} \subseteq F_*(\phi_1,e_2,1) = \{y_1,y_3\}$

and for $\phi_1, \phi_2 \in B$,

 $N^*((F,G,K)))=\{((\phi_1,e_2,1),\{y_1,y_3\}),((\phi_2,e_2,0),O)\}.$

Also, $Bnd_N(\sigma) \ge 0$ is satisfied.

Definition 3.3. Let $N(F_s, G_s, K)$ and $N(F_1, G_1, L)$ be NBSES over *O*. If

1. $K \subseteq L$,

2. For $F_s(\phi, e, o) \subseteq F_1(\phi, e, o)$ and $G_1(\neg \phi, e, o) \subseteq G_s(\neg \phi, e, o)$,

$$\not (\phi, e, o) \in K \subseteq P \times E \times O,$$

3. For $N_*(\sigma, K) = N_*(F_s(k), G_s, K)$ of a set (F_s, G_s, K) and $N_*(\mu, L) = N_*(F_1(\phi), G_1, L)$ of a set (F_1, G_1, L) ,

 $N_*(\sigma, K) \subseteq N_*(\mu, L),$

then, $N(F_s, G_s, K)$ is a near bipolar soft expert subset (NBSEs) of $N(F_1, G_1, L)$ and denoted by $N(F_s, G_s, K) \subseteq N(F_1, G_1, L)$.

Definition 3.4. If $N(F_s, G_s, K)$ is a NBSEs of $N(F_1, G_1, L)$ and $N(F_1, G_1, L)$ is a NBSEs of $N(F_s, G_s, K)$, then $N(F_s, G_s, K)$ and $N(F_1, G_1, L)$ are equal NBSES over O.

Definition 3.5. Let F^c and G^c be mappings where

$$\begin{split} F^{c}(\phi, e, 1) &= F(\phi, e, 0), F^{c}(\phi, e, 0) = F(\phi, e, 1), \\ G^{c}(\neg \phi, e, 1) &= G(\neg \phi, e, 0), G^{c}(\neg \phi, e, 0) = G(\neg \phi, e, 1) \end{split}$$

for $\forall \phi \in B$ and $\forall e \in E$. $N(F, G, K)^c = N(F^c, G^c, K)$ is a complement of a NBSES.

Definition 3.6. Let N(F, G, K) and $N(F_1, G_1, L)$ be two NBSES over O. The intersection of N(F, G, K) and $N(F_1, G_1, L)$ denoted by $N(H, I, M) = N(F, G, K) \cap N(F_1, G_1, L)$ where $\forall \phi \in B, \forall e \in E, \forall o \in o, M = K \cap L$,

 $N(H(\phi, e, o)) = \{ N(F(\phi, e, o) \cap F_1(\phi, e, o)), \quad if(\phi, e, o) \in K \cap L \},\$

 $N(I(\neg\phi, e, o)) = \{ N(G(\neg\phi, e, o) \cup G_1(\neg\phi, e, o)), \quad if(\phi, e, o) \in K \cap L \}$

and the union of N(F, G, K) *and* $N(F_1, G_1, L)$ *denoted by* $N(H, I, M) = N(F, G, K) \cup N(F_1, G_1, L)$ *where* $\forall \phi \in B, \forall e \in E, \forall o \in o, M = K \cap L$,

$$N(H(\phi, e, o)) = \{ N(F(\phi, e, o) \cup F_1(\phi, e, o)), \quad if(\phi, e, o) \in K \cap L \},\$$

 $N(I(\neg \phi, e, o)) = \{ N(G(\neg \phi, e, o) \cap G_1(\neg \phi, e, o)), \quad if(\phi, e, o) \in K \cap L \}.$

3.1. Application of Bipolar Soft Sets on Nearness Approximation Space

In this part, we will use the notion of near bipolar soft expert sets to make the best choice available to us. In order to do this, we will follow some steps. Let us now consider this with an example.

Example 3.7. Assume that a car selling firm has a set of cars O with a set of parameters \mathcal{F} . Let $O = \{y_1, y_2, y_3, y_4, y_5, y_6\}$ be a set of six car and $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3, ..., \phi_6\}$ be a set of six parameters, where ϕ_i , i = (1, 2, 3, 4, 5, 6) stand for inexpensive, speed, modern, fuel saving, engine capacity and acceleration, respectively. The equivalence classes obtained with the properties of the objects(ϕ_i , i = 1, 2, 3, 4) are given as follows:

$$[y_1]_{\phi_1} = \{y_1, y_3\}, [y_2]_{\phi_1} = \{y_2, y_4, y_5\}, [y_6]_{\phi_1} = \{y_6\}, \\ [y_1]_{\phi_2} = \{y_1, y_3\}, [y_2]_{\phi_2} = \{y_2, y_4\}, [y_5]_{\phi_2} = \{y_5\}, [y_6]_{\phi_2} = \{y_6\}.$$

We should noted that $\{\neg \phi_1, \neg \phi_2\}$ does denote expensive, slow, respectively. Now, assume that a car selling firm categorized these cars with interest to the set of parameters using a concept of a NBSES.

agree e _i	(ϕ_1, e_1)	$(-\phi_1, e_1)$	(ϕ_1, e_2)	$(\neg \phi_1, e_2)$	(ϕ_2, e_1)	$(\neg \phi_2, e_1)$	(ϕ_2, e_2)	$(\neg \phi_2, e_2)$	$P = \sum (\phi_i, e_i)$
<u>y</u> 1	1	0	1	0	1	0	1	0	4
<i>y</i> ₂	0	-1	0	-1	0	-1	0	-1	-4
<i>y</i> ₃	1	0	1	0	0	-1	1	0	2
<i>y</i> ₄	0	-1	0	-1	0	0	1	0	-1
¥5	0	-1	0	-1	0	0	0	-1	-3
¥6	1	0	0	-1	0	-1	0	-1	-2
disagree e _i	(ϕ_1, e_1)	$(-\phi_1, e_1)$	(ϕ_1, e_2)	$(\neg \phi_1, e_2)$	(ϕ_2, e_1)	$(\neg \phi_2, e_1)$	(ϕ_2, e_2)	$(\neg \phi_2, e_2)$	$R = \sum (\phi_i, e_i)$
y_1	0	-1	0	-1	0	-1	0	-1	-4
<i>y</i> ₂	1	0	1	0	1	0	1	-1	3
¥3	0	0	0	-1	0	-1	1	0	-1
¥4	1	0	0	0	0	-1	0	-1	-1
Y5	0	0	0	0	0	-1	1	0	0
¥6	0	-1	0	0	0	0	0	0	-1

Let $F : B \to 2^O$ and $G : -B \to 2^O$ be mappings and the equivalence classes obtained with the expert opinions about the properties of the objects are given as follows:

and

Now, suppose that we want to select a car with respect to $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F}$.

We determine the value of $(y_{\phi}, F(\phi_i))$ and $(y_{\phi}, G(-\phi_i))$ by the following two roles and construct table:

$$(y_n, F(\phi_i)) = \begin{cases} 1, & y_n \in F(\phi_i) \\ 0, & y_n \notin F(\phi_i) \end{cases}$$

•

If we combine it using $F(\phi_i) \cap G(\neg \phi_i) = \emptyset$ for each $\phi_i \in \mathcal{F}$, we get the following table.

$$(y_n, (F(\phi_i), G(-\phi_i))) = \begin{cases} 1, & y_n \in F(\phi_i) \\ -1, & y_n \in G(-\phi_i) \\ 0, & y_n \notin F(\phi_i) \cup G(-\phi_i) \end{cases}$$

Sort the values by the rule

$$Sumn = \sum_{i=1}^{6} (y_n, (F(\phi_i), G(\neg \phi_i)))$$

for which $d = \{\max Sumn : n = 1, 2, ..., s\}$, where $s = |y|, P = \sum (\phi_i, e_i)$ for agree e_i and $R = \sum (\phi_i, e_i)$ for disagree e_i . Then find the decision. d is the appropriately chosen car. If d has more than one value, any of them can be selected.

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decision	P	R	(P-R)
y_1	4	-4	8
y_2	-4	3	-7
<i>y</i> ₃	2	-1	3
y_4	-1	-1	0
y5	-3	0	-3
<u>y</u> ₆	-2	-1	-1

Let

$$N(F,G,K) = \{((\phi_1,e_1,1),\{y_1,y_3,y_6\}), ((\neg\phi_1,e_1,1),\{y_2\}), ((\phi_1,e_2,1),\{y_1,y_2,y_3\}), ((\neg\phi_1,e_2,0),\{y_5,y_4\})\},\$$

be BSES. Then N(F, G, K) is a NBSES. Because,

and

$$N^*(\sigma) = \{((\phi_1, e_1, \{y_1, y_3, y_6\}), (\phi_2, e_1, \{y_1, y_2, y_3, y_4\}))\} for\phi_1, \phi_2 \in B.$$

Now, it can be seen from the table that cars y_1 and y_2 are optimal cars with P - R values of 8 and 7, respectively. Therefore, choosing y_1 is appropriate to buy the car we want based on expert opinions. Accordingly, we find the most suitable car according to the ϕ_1 and ϕ_2 features based on expert opinions.

4. Conclusions

In this study, we classify the objects according to the new equivalence relation that we have obtained by considering the expert opinions and the properties of the objects in the concept of bipolar soft expert set. We got the concept. Thus, we have enabled us to see more clearly what we want, namely the practice of choosing the best products. We aim to obtain similar examples according to the definitions we will find in future studies.

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